Mathematics and Mathematics Learning Disorders:

Neuropsychological, Neurological, Cognitive and Psychometric Perspectives

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Proposition:

Number is a natural cognitive capacity of humans
Humans are born with a fundamental sense of quantity
Infant Cognitive Capacity and Number

- Evidence that infants possess some knowledge of
  - Numerosity
  - Ordinality
  - Addition and Subtraction
Preverbal timing and counting

- Preverbal counting based on timing
- Concept of number develops from here
Psychometric approach to individual differences in math ability

- Factor analytic studies
  - Two factors:
    1. Numerical facility
    2. Math reasoning
A child must come to understand

- uniqueness of each number -
  - represents a unique quantity

- seriality
  - successive numbers represent larger quantities

- each number represents groups of smaller numbers
  - basis of decomposing 8 into 6 plus 2

- use of numbers in other contexts
  - phone numbers, different meanings in vocabulary such as ‘second’
Numbers & Counting

Geary

- Developes preschool: ages 2-8
- natural development, but slow
Counting

- involves assigning word tags
- one-to-one correspondence
- manage irregularity (e.g., eleven)
Number Concepts

- mapping labels onto specific quantities
- usually past 4 years
- counting errors in partitioning or tagging
Typical Counting Errors
Cardinality & Ordinality

- word tags provide information
cardinality

• number-word assigned to last object represents the total
• perceptual cues can confuse child
ordinality

- successive number words represent larger quantities
- equivalence
- leads to less than, more than concepts
Developmental Mechanisms for Counting and Number Knowledge

see Gelman and Gallistell, 1978

- counting behavior is guided by 5 implicit principles
  - one-to-one correspondence
  - stable order
  - cardinality
  - abstraction
  - order irrelevance
- 1st three are ‘how-to’
- other theories
Representation of numbers and numerical information
(McCloskey, 1995)

- basic forms of number representation:
  - verbal numeral
  - phonological
  - graphemic
  - Arabic numeral
- basic forms must get transformed into semantic representations
- disagreement about how facts are stored and retrieved
The Neuropsychology of Math

Math Abilities and Disabilities
Hemispheric Specialization

- EEG evidence of left hem specialization for sequential, timed info: auditory
- Right hem specialization for global (simultaneous) info: visual
Localization of Math Abilities

Right hem  vis-spatial org.
Lang. Dom. hem.  linguistic skills
Hi assoc. dom. hem  word probs; math concepts, proc.
Frontal  quick calc., abstrct, prob solve, writ. oral
Parietal  motor, tactile
Left parietal  sequencing
Occipital  visdiscrim symb.
Temporal  auθercep; L-T αθerm
Dom. temporal  mem of series, math facts, subvoc
Hegerty’s study of spatial involvement (1999):

2 types of imagery: visual & spatial

- **Visual Imagery** - pictorial representations
  - a representation of the visual appearance of the object
  - object representation, including shape, color, brightness, fxs subserved by ventral stream and parvocellular system
Hegerty; spatial imagery -

- schematic representations
- spatial relationships between parts of an object and location of object in space, movement,
  - subserved by dorsal stream and magnocellular system
Schematic representations more effective

- constructed tasks to assess use of these 2 functions in solving math problems (6th grade boys)
- use of schematic/spatial representations was positively related to success in solving math problems
  - pictorial representations was negatively related to success
  - Block Design was the test most highly correlated with math scores
2 visual streams
Left inferior parietal lobule plays role in calculation.
Many studies confirm

- PET
- fMRI
- ERP

But questions about this area in the exact role of calculation
Sources of mathematical thinking
(Dehaene et al, 1999)

- In processing elementary arithmetic, most propose 2 representational formats for number:
  - Language-based format for exact arithmetic knowledge
  - Language-independent representation of number magnitude
    - Number line
    - Quantity manipulation
    - Approximation
  - See McCloskey for disagreement
Dissociation between calculation and approximation

- Calculation is language dependent.
- Approximation relies on visuo-spatial networks.
The Triple-Code Model of Number Processing

- L & R inferior parietal cortices contain analogical representation of numerical quantity
  - Compares numbers & performs simple quantity manipulations/approximations

- Calculations that require manipulations of verbal representations of numbers
  - Engage perisylvian language areas
  - I.e., retrieving result of multiplication in rote verbal memory
Dehaene fMRI studies

- bilateral parietal lobes showed greater activation for approximation than calculation
- areas involved in visuospatial and analogical transformations
- most areas activated fell outside language areas (perisylvian)
- Calculation revealed a left-lateralized pattern (left inferior frontal lobe)
Is parietal activation relative to numbers in basic processing?

- Stanescu-Cosson (2000) controlled for number specific functions
- Assert that the parietal activations for number processing are most likely due to
  - Attention
  - Visual-spatial coordinate transformations
- Thus, no evidence that internal manipulation of numbers draws on more than general visuo-spatial resources
But, intraparietal activation important for approximation

- Stanescu-Cosson confirmed Dehaene dissociation
- Angular gyri became involved in exact calculations
  - Angular gyrus of left hem participate in language
  - Part of perisylvian region
Angular Gyrus
MATH DISABILITIES:

Studies of Arithmetic Disabilities
Neuropsychological Approaches

- considerable evidence of neurological involvement in learning disorders
- studies of acquired and developmental dyscalculias converge on same conclusions
  - remember: results of studies are not clear cut
  - specific lesions do not always produce specific deficits
Acalculia

- acalculia (no counting)
  - appreciation of knowledge of number concepts (acalculia with verbal deficits)
  - ability to organize and manipulate numbers spatially (long division, mult. of 2+ numbers) (spatial dyscalculia)
  - ability to perform arithmetic operations (anarithmetria)
Three Categories of Dyscalculia

- Neuropsychological dysfunction/diagnoses
  - Compare with Math Disability
    1. alexia and agraphia for numbers
    2. spatial acalculia
    3. anarithmetria
(4. Badian: attentional-sequential dyscalculia)
Alexia & Agraphia for Numbers

- difficulties in reading & writing numbers
- intact arithmetic processes
- typically left hemisphere
  - sometimes assoc. with aphasia
- rare in children
Spatial Acalculia

- difficulty in spatial representation of numerical information
- also conceptual understanding of associated representations (e.g., place value)
- number reading & writing, basic computation (fact retrieval) intact
Spatial Acalculia
Anatomical Localization: Structural Deficits

- associated with posterior right hemisphere damage
  - misalignment of numbers in columns
  - number omission
  - number rotation
  - misread signs
  - difficulty with place value & decimals
Developmental Considerations

- visuospatial deficits might impact development of basic skills
- use of visuospatial representation for counting & solutions before they are automatized
- studies suggest more a problem for boys
Anarithmetria

- retrieval of basic facts from LT memory
- some verbal deficits
- some difficulty with operations and procedures (e.g., algorithms; $0 \times N=0$, oral problems and carrying)
- posterior left hemisphere damage
- Ashcraft: early left lesion group had trouble retrieving facts, unsystematic in solutions
- number reading & writing, concepts intact
Badian’s Attention Sequence

• general attention deficits
• difficulties with serial execution of arithmetic operations
• prefrontal, typically left hemisphere
Developmental Dyscalculia

Based on McCloskey

- Number comprehension
- Number production
- Calculation
Number comprehension

- Correspondence of number to quantity
- More, less, equal concepts
- Serial order
Number production

- Counting
- Writing & reading numbers
Calculation

- Use of operation signs
- Overlearned number facts
- Simple and complex operations
Cognitive Studies

- Basic lower order cognitive numerical skills
  - representation and retrieval of basic arithmetic facts from LT memory
  - Metacognitive & executive skills important
    - affects performance in many errors
Experimental studies in counting and arithmetic skills

MD children show 2 functional (phenotypic) numerical deficits

1. developmentally immature arithmetic procedures & high frequency of procedural errors
2. difficulty in representation and retrieval of facts from long-term semantic memory
1. developmentally immature arithmetic procedures & hi freq. of procedural errors

- developmental delay in acquisition of conceptual knowledge underlying procedure use
- possible working memory contribution
2. difficulty in representation and retrieval of facts from LT semantic memory

- more fundamental: does not disappear with development
- use of strategies for solving problems if answer not available from LT memory
Strategy development

- changing the mix of existing strategies & constructing new ones
  - abandon old
- e.g., count on fingers, verbal counting, direct retrieval
mastery of elementary arithmetic is achieved when all basic facts can be retrieved from LT memory without error. This appears to facilitate acquisition of more complex skills.
Development of Memory Representation

- connection between problems and answers
- leads to direct retrieval
- based on execution of computational strategies
- each execution of computational strategy increases probability of direct retrieval of subsequent solutions
Working Memory

- for retrieval from LT memory, several things must be going on
  - augend (1st number)
  - addend (2nd number)
  - answer,
  - all must be active in working memory
- also how much info can be rehearsed in 2-3 sec span
  - span memory or phonological loop
  - rehearsal time appears related to counting speed
Development of LT memory representations

- of basic arithmetic facts
- related to speed of executing computational strategies
- AND accuracy
- both related to memory span
- memory retrieval deficits appear to persist through elementary school
"I asked you a question, buddy. ... What's the square root of 5,248?"
Arithmetic Performance and Achievement

- Counting knowledge
- Working memory
  - Attention allocation
- Working memory
  - Decay rate
- Counting speed

- Procedural skill
- Knowledge base
  - Fact retrieval

- Test performance
Taxonomy of MD

- 3 general subtypes
  - Compare with dyscalculias
  1. Fact retrieval: Semantic Memory
  2. Procedural
  3. Visuospatial
Semantic Memory: Cognitive & Performance Features

- low frequency of arithmetic fact retrieval
- when retrieved, high error rate
- solution times for correct retrieval unsystematic
Semantic Memory: Neuropsychological Features

- associated with left hemisphere dysfunction
  - posterior
- possible subcortical involvement
  - thalamus
Semantic Memory: Genetics & Relationship to RD

♠ unclear genetic relationship
♠ associated with certain forms of RD suggests deficit is heritable
♠ often covaries with RD
   ✦ especially phonetic deficits
Procedural: Cognitive & Performance Features

- frequent use of developmentally immature procedures
  - E.g., over use of counting
- errors in execution of procedures
- potential delay understanding concepts of procedural use
Procedural: Neuropsychological Features

- unclear - left hemisphere?
- Genetics & relationship with RD also unclear
Fact retrieval & procedural skills

- some evidence that these are dissociable/independent
- both associated with left-hem lesion (Ashcraft, 1992)
- Anarithmetric deficits same as procedural & memory-retrieval deficits in cognitive research
Visuospatial: Cognitive & Performance Features

- more typical of boys than girls

- difficulty spatially representing numerical info
  - misalignment of numbers in columns, rotating numbers

- misinterpretation of spatially represented numerical info
  - place value errors

- Difficulty estimating (?)
Visuospatial: Neuropsychological Features

- associated with right hemisphere dysfunction
  - posterior
Visuospatial: Genetic & Relationship with RD

- genetic features unclear
- apparently not associated with phonologically based RD
Summary of Neuropsychological Studies

- 3 distinct lower order math deficits
  - fact retrieval
  - procedural
  - spatial representation
Comparison with DC

Cognitive MD
- fact retrieval
- procedural
- spatial representation

Dev. Dyscalc.
- number comprehension
- number production
- calculation
Fact retrieval deficits often co-occur with some language-reading disabilities (spatial do not)

- some studies: procedural & reading deficits co-occur
- do verbal skills mediate initial learning of procedures which later become routinized?
Relationship between RD & MD

- arithmetic facts represented in associative memory network
- retrieval of arith. facts from LT memory similar to verbal retrieval (semantic memory)
- RD have difficulties associated with auditory memory
Posterior Left Hemisphere

- fact retrieval correlated with reading skill (Geary, 1991)
- executing computation procedures did NOT correlate with reading skill
- fact retrieval deficits frequently occur with damage to L-posterior (acquired dyscalculia)
- much evidence of L-posterior damage in dyslexia
Co-Ocurrence of MD & RD

- common underlying deficit *might* cause RD-MD to co-occur
  - at cognitive level: difficulty in representation & retrieval of semantic info from LT memory
Genetic component

- no clear associations, although suspected
- MD & RD co-exist in many
  - RD has a frequent genetic component


